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Superfluidity of 'dirty' indirect magnetoexcitons in coupled quantum wells in high magnetic field

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Abstract

We present a theory of superfluidity in the quasi-two-dimensional system of spatially indirect magnetoexcitons in coupled quantum wells in the presence of a random field. The problem of a dilute gas of magnetoexcitons with dipole–dipole repulsion in the limit of high magnetic field can be mapped to the problem of a dilute gas of two-dimensional excitons in a random field without magnetic field. The density of the superfluid component n_s and the critical temperature T_c of the Kosterlitz–Thouless transition to a superfluid state are obtained as functions of magnetic field H, electron–hole spatial separation D, and the random field parameters. For two-dimensional magnetoexciton systems, increasing the magnetic field H and the distance D suppresses the superfluid density and the critical temperature of the Kosterlitz–Thouless transition. The influence of the interwell distance D on n_s and T_c in strong magnetic field is opposite to the case without magnetic field, where n_s and T_c increase with increasing D, for fixed total exciton density n. We show that in the presence of the disorder there must be a quantum phase transition from a superfluid to a disordered phase at $T = 0$ as the magnetic field H is varied. There is no superfluidity at any exciton density in the presence of the disorder at sufficiently large magnetic field H or sufficiently large disorder.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Systems of excitons with spatially separated, two-dimensional (2D) electrons and holes, known as indirect excitons, in magnetic fields (H) have been the subject of several experimental investigations [1–4]. Interest has been further spurred by the recent success in producing

structures with very long exciton lifetime and diffusion length [5, 6], which allows the possibility of equilibration in a macroscopic trap. Indirect excitons are of interest, in particular, in connection with the possibility of excitonic superfluidity, which could manifest itself as persistent electrical currents in coupled quantum wells (CQW) [7–13]. Substantial theoretical work has also been carried out for the simpler system of excitons in single quantum wells in high magnetic fields [14–24]. A general consequence of high magnetic field is that the mass of the exciton does not depend on the band electron and hole masses, but instead, for H greater than about 7 T, the exciton mass depends only on the strength of the magnetic field. In this limit the binding energy of the excitons increases with increasing magnetic field, which means that magnetoexcitons can survive in a substantially wider temperature region than the excitons at zero magnetic field. Lerner and Lozovik [14] and Paquet *et al* [15] predicted that under some conditions of symmetry in the system, exact solutions of the many-body problem in the Hartree–Fock limit correspond to ideal Bose condensation of magnetoexcitons at *any* density. Lerner and Lozovik later showed [16] that including the polarizability of the excitons restores a weak repulsive interaction between the excitons, so that they can be treated as a weakly interacting Bose gas. Some novel predictions include the existence of a dipolar supercurrent in electron–hole bilayer systems which can be tuned by an in-plane magnetic field [17] and the existence of a stable dielectric liquid state which is also superfluid [24].

In the following, we assume a translationally invariant two-dimensional system; the superfluid state appears in the system under consideration below the temperature of Kosterlitz–Thouless transition [25]. This was studied recently [13] for systems with spatially separated electrons and holes in the absence of magnetic field, including the effect of disorder [26].

The critical prediction of interest to experiments is the temperature of the phase transition to a superfluid state. It was shown [21] that increasing the magnetic field at a fixed magnetoexciton density leads to a reduction of the Kosterlitz–Thouless transition temperature T_c on account of the increase of the exciton magnetic mass as a function of a magnetic field H. But it turns out that the highest possible Kosterlitz–Thouless transition temperature increases with H (at small D), due to the rise in the maximal density of magnetoexcitons as a function of H [21].

Previous theory of superfluidity in magnetoexcitonic systems did not take into account the role of disorder, which is created by impurities and boundary irregularities of the quantum wells. In real experiments, however, disorder plays a very important role. Although the inhomogeneous broadening linewidth of typical GaAs-based samples has been improved from around 20 meV to less than 1 meV [22], the disorder energy is still not much smaller than the exciton–exciton repulsion energy at typical exciton densities. At an exciton density of 10^{10} cm^{-2}, the interaction energy of the excitons is approximately 1 meV [27]. On the other hand, the typical disorder energy of 1 meV is low compared to the typical exciton binding energy of 5 meV.

A typical experimental example of indirect excitons in coupled quantum wells is two coupled GaAs quantum wells with AlGaAs barriers [5, 6]. Fluctuations of the thickness of a quantum well, which arise during the fabrication process, impurities in the system, and disorder in the alloy of the barriers can all lead to the appearance of a random field. The dominant type of disorder at low temperature is believed to be fluctuation of the barrier alloy composition, with a characteristic length scale short compared to the excitonic Bohr radius of around 100 Å.

This paper joins together two lines of theoretical investigation which up to now have been pursued separately: the theory of magnetoexciton condensates without disorder, and the theory of superfluids in the presence of disorder. The collective properties and superfluidity of 2D weakly interacting indirect excitons in weak disorder without magnetic field were analyzed in [28]. In [26], this theory was generalized to the case of a random field which is not

necessarily small compared to the dipole–dipole repulsion between excitons for CQW, again, in the absence of magnetic field. The second-order Born approximation allowed us to derive analytically the 2D indirect exciton Green's function in the weak scattering limit (the second-order Born approximation Green's function for 3D excitons was obtained by Gevorkyan and Lozovik [29]). It was predicted that in the low temperature limit, the density n_s of the superfluid component in CQW systems and the temperature of the superfluid transition (the Kosterlitz–Thouless temperature T_c [25]) decrease with increasing strength of the random field [26].

Some earlier theory has treated the case of magnetoexcitons in the presence of disorder, without including the effect of superfluidity. The absorption spectra and wavefunctions of optically active single magnetoexciton states in disordered quantum wells were calculated numerically by Grochol *et al* [27]. In the fermion description the single-particle Green's function and the kernel of the Bethe–Salpeter equation both depend on the random potential and the Coulomb interaction between the electrons and holes, and so the disorder and the interactions were treated simultaneously on the level of the Bethe–Salpeter equation. In [30], the spectrum of a single indirect 2D magnetoexciton (noninteracting with other magnetoexcitons) in a strong perpendicular magnetic field in the presence of disorder was obtained in the second-order Born approximation.

In the present paper we derive analytically the collective spectrum of the 2D low density gas of weakly interacting indirect magnetoexcitons in the presence of the disorder, including the dipole–dipole repulsion between magnetoexcitons in the ladder approximation [21]. We consider disorder which is not weak compared to the dipole repulsion in the second-order Born approximation [30]. The density of the superfluid component n_s and the temperature T_c of the Kosterlitz–Thouless transition to a superfluid state are obtained as functions of magnetic field H, interlayer separation D and the random field parameters α_i and g_i. We show that the density of the superfluid component and the Kosterlitz–Thouless temperature of the superfluid phase transition decrease as the random field increases. These results are derived by mapping the problem of the dilute indirect magnetoexciton gas with disorder in a strong magnetic field onto the problem of the dilute dipole gas without magnetic field, consisting of indirect magnetoexcitons with an effective mass which is a function of the magnetic field and the parameters of the coupled quantum wells (CQW), in an H-dependent effective random field. The application of these results, obtained for a CQW with spatially separated electrons and holes, is also discussed for the case of an unbalanced two-layer electron system.

The paper is organized in the following way. In section 2 we derive the effective Hamiltonian of dilute, 'dirty', spatially indirect magnetoexcitons in the effective magnetic mass approximation in the high magnetic field limit. In section 3 we obtain the Green's function of a single magnetoexciton in the random field. In section 4 the collective spectrum, the superfluid density and the temperature of the Kosterlitz–Thouless phase transition of magnetoexcitons are derived in the presence of disorder. In section 5 we discuss our results and consider the extension of the framework used for the indirect magnetoexcitons in CQW on the system of indirect magnetoexcitons in unbalanced two-layer electron system.

2. The effective Hamiltonian of dilute 'dirty' spatially indirect magnetoexcitons in the effective magnetic mass approximation

The total Hamiltonian $\hat{H}$ of 2D spatially separated e and h in the perpendicular magnetic field in the presence of the external field in the second quantization representation has the form ($\hbar = c = 1$):

$$\hat{H} = \int \mathrm{d}\mathbf{R} \int \mathrm{d}\mathbf{r} \Bigg[\hat{\psi}^{\dagger}(\mathbf{R},\mathbf{r}) \Bigg(\frac{1}{2m_{\mathrm{e}}} (-\mathrm{i}\nabla_{\mathrm{e}} + e\mathbf{A}_{\mathrm{e}})^2 + \frac{1}{2m_{\mathrm{h}}} (-\mathrm{i}\nabla_{\mathrm{h}} - e\mathbf{A}_{\mathrm{h}})^2 - \frac{e^2}{\epsilon\sqrt{(\mathbf{r}_{\mathrm{e}} - \mathbf{r}_{\mathrm{h}})^2 + D^2}} + V_{\mathrm{e}}(\mathbf{r}_{\mathrm{e}}) + V_{\mathrm{h}}(\mathbf{r}_{\mathrm{h}}) \Bigg) \hat{\psi}(\mathbf{R},\mathbf{r}) \Bigg] + \tfrac{1}{2} \int \mathrm{d}\mathbf{R}_1 \int \mathrm{d}\mathbf{r}_1 \int \mathrm{d}\mathbf{R}_2 \int \mathrm{d}\mathbf{r}_2 \hat{\psi}^{\dagger}(\mathbf{R}_1,\mathbf{r}_1) \hat{\psi}^{\dagger}(\mathbf{R}_2,\mathbf{r}_2) \times \left(U^{\mathrm{ee}}(\mathbf{r}_{\mathrm{e}1} - \mathbf{r}_{\mathrm{e}2}) + U^{\mathrm{hh}}(\mathbf{r}_{\mathrm{h}1} - \mathbf{r}_{\mathrm{h}2}) + U^{\mathrm{eh}}(\mathbf{r}_{\mathrm{e}1} - \mathbf{r}_{\mathrm{h}2}) + U^{\mathrm{he}}(\mathbf{r}_{\mathrm{h}1} - \mathbf{r}_{\mathrm{e}2}) \right) \times \hat{\psi}(\mathbf{R}_2,\mathbf{r}_2) \hat{\psi}(\mathbf{R}_1,\mathbf{r}_1). \tag{1}$$

Here $\hat{\psi}^{\dagger}(\mathbf{R},\mathbf{r})$ and $\hat{\psi}(\mathbf{R},\mathbf{r})$ are the operators of creation and annihilation of magnetoexcitons; $\mathbf{r}_{\mathrm{e}}$ and $\mathbf{r}_{\mathrm{h}}$ are electron and hole locations along quantum wells, respectively; $\mathbf{A}_{\mathrm{e}}$, $\mathbf{A}_{\mathrm{h}}$ are the vector potentials in the electron and hole locations, respectively; $V_{\mathrm{e}}(\mathbf{r}_{\mathrm{e}})$ and $V_{\mathrm{h}}(\mathbf{r}_{\mathrm{h}})$ represent the external fields acting on electron and hole, respectively (we use units $c = \hbar = 1$); D is the distance between electron and hole quantum wells; e is the charge of an electron; ϵ is the dielectric constant. We use below the coordinates of the magnetoexciton center of mass $\mathbf{R} = (m_{\mathrm{e}}\mathbf{r}_{\mathrm{e}} + m_{\mathrm{h}}\mathbf{r}_{\mathrm{h}})/(m_{\mathrm{e}} + m_{\mathrm{h}})$ and the internal exciton coordinates $\mathbf{r} = \mathbf{r}_{\mathrm{e}} - \mathbf{r}_{\mathrm{h}}$. The cylindrical gauge for vector potential is used: $\mathbf{A}_{\mathrm{e,h}} = \frac{1}{2}\mathbf{H} \times \mathbf{r}_{\mathrm{e,h}}$. U^{ee}, U^{hh}, U^{eh} and U^{he} are the two-particle potentials of the electron–electron, hole–hole, electron–hole and hole–electron interaction, respectively, between electrons or holes from different pairs:

$$\begin{aligned} U^{\mathrm{ee}}(\mathbf{r}_{\mathrm{e}1} - \mathbf{r}_{\mathrm{e}2}) &= \frac{e^2}{\epsilon|\mathbf{r}_{\mathrm{e}1} - \mathbf{r}_{\mathrm{e}2}|}; \qquad U^{\mathrm{hh}}(\mathbf{r}_{\mathrm{h}1} - \mathbf{r}_{\mathrm{h}2}) = \frac{e^2}{\epsilon|\mathbf{r}_{\mathrm{h}1} - \mathbf{r}_{\mathrm{h}2}|}; \\ U^{\mathrm{eh}}(\mathbf{r}_{\mathrm{e}1} - \mathbf{r}_{\mathrm{h}2}) &= -\frac{e^2}{\epsilon\sqrt{|\mathbf{r}_{\mathrm{e}1} - \mathbf{r}_{\mathrm{h}2}|^2 + D^2}}; \\ U^{\mathrm{he}}(\mathbf{r}_{\mathrm{h}1} - \mathbf{r}_{\mathrm{e}2}) &= -\frac{e^2}{\epsilon\sqrt{|\mathbf{r}_{\mathrm{h}1} - \mathbf{r}_{\mathrm{e}2}|^2 + D^2}}. \end{aligned} \tag{2}$$

The conserved quantity for an isolated exciton in magnetic field without any external field ($V_{\mathrm{e}}(\mathbf{r}_{\mathrm{e}}) = V_{\mathrm{h}}(\mathbf{r}_{\mathrm{h}}) = 0$) is the exciton *magnetic* momentum (see [31]):

$$\hat{\mathbf{P}} = -\mathrm{i}\nabla_{\mathrm{e}} - \mathrm{i}\nabla_{\mathrm{h}} + e(\mathbf{A}_{\mathrm{e}} - \mathbf{A}_{\mathrm{h}}) - e\mathbf{H} \times (\mathbf{r}_{\mathrm{e}} - \mathbf{r}_{\mathrm{h}}). \tag{3}$$

The conservation of this quantity is related to the invariance of the system upon a simultaneous translation of e and h and gauge transformation.

The eigenfunctions of the Hamiltonian of a single isolated magnetoexciton without any random field ($V_{\mathrm{e}}(\mathbf{r}_{\mathrm{e}}) = V_{\mathrm{h}}(\mathbf{r}_{\mathrm{h}}) = 0$), which are also the eigenfunctions of the magnetic momentum $\mathbf{P}$, have the following form (see [14, 31]):

$$\Psi_{k\mathbf{P}}(\mathbf{R},\mathbf{r}) = \exp\left\{ \mathrm{i}\mathbf{R}\left(\mathbf{P} + \frac{e}{2}\mathbf{H} \times \mathbf{R} \right) + \mathrm{i}\gamma\frac{\mathbf{P}\mathbf{r}}{2} \right\} \Phi_k(\mathbf{P},\mathbf{r}), \tag{4}$$

where $\Phi_k(\mathbf{P},\mathbf{r})$ is the function of internal coordinates $\mathbf{r}$; $\mathbf{P}$ is the eigenvalue of magnetic momentum; k are quantum numbers of j exciton internal motion. In high magnetic fields $k = (n_L, m)$, where $n_L = \min(n_1, n_2)$, $m = |n_1 - n_2|$, and n_1 and n_2 are the Landau quantum numbers for e and h [14, 20]; $\gamma = (m_{\mathrm{h}} - m_{\mathrm{e}})/(m_{\mathrm{h}} + m_{\mathrm{e}})$.

In this section we reduce the problem of the dilute gas of magnetoexcitons (in high magnetic field) with dipole–dipole repulsion in a random field to the problem of the dilute gas of dipole excitons without magnetic field with a new effective magnetic mass of the exciton, which is a function of the magnetic field and the parameters of the quantum wells, in an H-dependent effective random field. We assume the excitons all lie in the lowest Landau level (the high magnetic field regime).

We expand the magnetoexciton field operators in a single-magnetoexciton basis set $\Psi_{k\mathbf{P}}(\mathbf{R}, \mathbf{r})$

$$\hat{\psi}^{\dagger}(\mathbf{R}, \mathbf{r}) = \sum_{k\mathbf{P}} \Psi^{*}_{k\mathbf{P}}(\mathbf{R}, \mathbf{r})\hat{a}^{\dagger}_{k\mathbf{P}}; \qquad \hat{\psi}(\mathbf{R}, \mathbf{r}) = \sum_{k\mathbf{P}} \Psi_{k\mathbf{P}}(\mathbf{R}, \mathbf{r})\hat{a}_{k\mathbf{P}}, \tag{5}$$

where $\hat{a}^{\dagger}_{k\mathbf{P}}$ and $\hat{a}_{k\mathbf{P}}$ are the corresponding creation and annihilation operators of a magnetoexciton in $(k, \mathbf{P})$ space.

We consider the case of strong magnetic field, when we neglect in equation (4) the transitions between different Landau levels of the magnetoexciton caused by scattering by the potential $V_{\mathrm{e}}(\mathbf{r}_{\mathrm{e}}) + V_{\mathrm{h}}(\mathbf{r}_{\mathrm{h}})$. We also neglect nondiagonal matrix elements of the Coulomb interaction between electron and hole in the same pair. The application region of these two assumptions is defined by the inequalities

$$\omega_{\mathrm{c}} \gg E_{\mathrm{b}}, \qquad \omega_{\mathrm{c}} \gg \sqrt{\left\langle V^{2}_{e(h)} \right\rangle_{\mathrm{av}}}, \tag{6}$$

where $\omega_{\mathrm{c}} = \sqrt{eH/m_{\mathrm{e-h}}}$, $m_{\mathrm{e-h}} = m_{\mathrm{e}}m_{\mathrm{h}}/(m_{\mathrm{e}} + m_{\mathrm{h}})$ is the exciton reduced mass in the quantum well plane; E_{b} is the magnetoexciton binding energy in an ideal 'pure' system as a function of magnetic field H and the distance between electron and hole quantum wells D: $E_{\mathrm{b}} \sim e^{2}/\epsilon r_{\mathrm{H}}\sqrt{\pi/2}$ at $D \ll r_{\mathrm{H}}$ and $E_{\mathrm{b}} \sim e^{2}/\epsilon D$ at $D \gg r_{\mathrm{H}}$ ($r_{\mathrm{H}} = (eH)^{-1/2}$ is the magnetic length) [14, 20]. Here $\langle \ldots \rangle_{\mathrm{av}}$ denotes averaging over the fluctuations of random field.

After exciton–exciton scattering the total magnetic momentum $\mathbf{P} = \sum_i \mathbf{P}_i$ is conserved, but the magnetic momentum $\mathbf{P}_i$ of each exciton can be changed. Since in a strong magnetic field, the mean distance $\boldsymbol{\rho}$ between the electron and hole in the plane of the quantum wells is proportional to the magnetic momentum ($\boldsymbol{\rho} = \frac{r_{\mathrm{H}}^{2}}{H}\mathbf{H} \times \mathbf{P}$) [14, 31], the scattering is accompanied by the exciton polarization. We consider sufficiently low temperatures when magnetoexciton states with only small magnetic momenta $P \ll \frac{1}{r_{\mathrm{H}}}$ are filled. The change of these magnetic momenta due to exciton–exciton scattering is also negligible due to the conservation of the total magnetic momentum. But these small magnetic momenta correspond to small separation between electrons and holes in the plane of the quantum wells $\rho \ll r_{\mathrm{H}}$. Therefore, the magnetoexciton polarization due to scattering is negligible and the magnetoexciton dipole moment stays almost normal to the quantum wells, $d = eD$, i.e. the interexciton interaction law is not changed due to the scattering. For the dilute system in the strong magnetic field $n \ll r_{\mathrm{H}}^{-2}$ (n is the 2D density of excitons, and the characteristic radius of magnetoexciton in the plane of the wells in strong magnetic field at small P approximately equals the magnetic length r_{H} [20]) the exciton–exciton interaction is a dipole–dipole repulsion, because the average distance between excitons r_{s} is large compared to the exciton radius ($r_{\mathrm{s}} = (\pi n)^{-1/2} \gg r_{\mathrm{H}}$). Therefore for the dilute system in a strong magnetic field at $n \ll r_{\mathrm{H}}^{-2}$ we have:

$$\hat{U}(|\mathbf{R}_1 - \mathbf{R}_2|) \equiv \hat{U}^{\mathrm{ee}} + \hat{U}^{\mathrm{hh}} + \hat{U}^{\mathrm{eh}} + \hat{U}^{\mathrm{he}} \simeq \frac{e^{2}D^{2}}{\epsilon|\mathbf{R}_1 - \mathbf{R}_2|^{3}}. \tag{7}$$

Notice that the magnetoexcitons repel like parallel dipoles, and their pair interaction potential depends only on the coordinates of the center of mass of the excitons and does not depend on the coordinates of the relative motion of the electron and hole.

Now we substitute the expansions (5) for the field creation and annihilation operators in the total Hamiltonian (1) and obtain the effective Hamiltonian in terms of creation and annihilation operators in $\mathbf{P}$ space. In strong magnetic fields, $\omega_{\mathrm{c}} = eH/\mu^{*} \gg e^{2}/r_{\mathrm{H}}$, the characteristic value of e–h separation in the magnetoexciton $|\langle \mathbf{r} \rangle|$ has the order of the magnetic length $r_{\mathrm{H}} = 1/\sqrt{eH}$. The functions $\Phi_k(\mathbf{P}, \mathbf{r})$ (see equation (4)) are dependent on the difference $(\mathbf{r} - \boldsymbol{\rho})$, where $\boldsymbol{\rho} = \frac{r_{\mathrm{H}}^{2}}{H}\mathbf{H} \times \mathbf{P}$ [14, 31]. At small magnetic momenta $P \ll 1/r_{\mathrm{H}}$ we

have $\rho \ll r_{\rm H}$, and, therefore, in the functions $\Phi_k(\mathbf{r}-\boldsymbol{\rho})$ we can ignore the variable $\boldsymbol{\rho}$ in comparison with $\mathbf{r}$. In strong magnetic fields, the quantum numbers k correspond to the quantum numbers (m, n_L) (see above). For the lowest Landau level we denote the spectrum of the single exciton $\varepsilon_0(P) \equiv \varepsilon_{00}(\mathbf{P})$. In high magnetic field, when the typical interexciton interaction $D^2 n^{-\frac{3}{2}} \ll \omega_{\rm c}$, one can ignore transitions between Landau levels and consider only the states in the lowest Landau level $m = n_L = 0$. Using the orthonormality of functions $\Phi_{mn}(\mathbf{0}, \mathbf{r})$ we obtain the effective Hamiltonian $\hat{H}_{\rm eff}$ in strong magnetic fields. Since a typical value of r is $r_{\rm H}$, and $P \ll 1/r_{\rm H}$ in this approximation, the effective Hamiltonian $\hat{H}_{\rm eff}$ in the magnetic momentum representation P in the lowest Landau level $m = n_L = 0$ has the same form (cf [13]) as that of a two-dimensional boson system without magnetic field, but with the magnetoexciton magnetic mass $m_{\rm H}$ (which depends on H and D) instead of the exciton mass ($M = m_{\rm e} + m_{\rm h}$), magnetic momenta instead of ordinary momenta, and the renormalized random field:

$$\hat{H}_{\rm eff} = \sum_{\mathbf{P}} \varepsilon_0(P) \hat{a}^\dagger_{\mathbf{P}} \hat{a}_{\mathbf{P}} + \sum_{\mathbf{P},\mathbf{P}'} \langle \mathbf{P}'|\hat{V}|\mathbf{P}\rangle \hat{a}^\dagger_{\mathbf{P}'} \hat{a}_{\mathbf{P}} + \tfrac{1}{2} \sum_{\mathbf{P}_1,\mathbf{P}_2,\mathbf{P}_3,\mathbf{P}_4} \langle \mathbf{P}_1, \mathbf{P}_2|\hat{U}|\mathbf{P}_3, \mathbf{P}_4\rangle \hat{a}^\dagger_{\mathbf{P}_1} \hat{a}^\dagger_{\mathbf{P}_2} \hat{a}_{\mathbf{P}_3} \hat{a}_{\mathbf{P}_4}, \quad (8)$$

where $\hat{V} = \hat{V}_{\rm e} + \hat{V}_{\rm h}$. The dispersion relation $\varepsilon_0(P)$ of an isolated magnetoexciton on the lowest Landau level is the quadratic function at small magnetic momenta under consideration:

$$\varepsilon_0(\mathbf{P}) \approx \frac{P^2}{2m_{\rm H}}, \quad (9)$$

where $m_{\rm H}$ is the effective *magnetic* mass of a magnetoexciton on the lowest Landau level, dependent on H and the distance D between the electron and hole planes (see [20]). In strong magnetic fields, with $D \gg r_{\rm H}$, the exciton magnetic mass is $m_{\rm H} \approx D^3\epsilon/(e^2 r_{\rm H}^4)$ [20]. The quadratic dispersion relation (9) is true for small P at arbitrary magnetic fields H and follows from the fact that $P = 0$ is an extremal point of the dispersion law $\varepsilon_k(\mathbf{P})$. The last statement may be proved by taking into account the regularity of the effective Hamiltonian $H_{\mathbf{P}}$ as a function of the parameter $\mathbf{P}$ at $\mathbf{P} = 0$ and also the invariance of $H_{\mathbf{P}}$ upon simultaneous rotation of $\mathbf{r}$ and $\mathbf{P}$ in the CQW plane. For the magnetoexciton ground state, $m_{\rm H} > 0$. For high magnetic fields $r_{\rm H} \ll a_0^*$ and at $D \lesssim r_{\rm H}$, the quadratic dispersion relation is valid at $P \ll 1/r_{\rm H}$, but for $D \gg r_{\rm H}$ it holds over a wider region—at least at $P \ll D/r_{\rm H}^2$ [20] ($a_0^* = \epsilon/(2m_{\rm e-h}e^2)$ is the radius of a 2D exciton at $H = 0$).

The matrix element of the inter-magnetoexciton interaction $\langle \mathbf{P}_1, \mathbf{P}_2|\hat{U}|\mathbf{P}_3, \mathbf{P}_4\rangle$ is defined as

$$\langle \mathbf{P}_1, \mathbf{P}_2|\hat{U}|\mathbf{P}_3, \mathbf{P}_4\rangle = \frac{1}{S^2} \int {\rm d}^2 R_1 \int {\rm d}^2 R_2 \int {\rm d}^2 r_1 \int {\rm d}^2 r_2 U(\mathbf{R}_1 - \mathbf{R}_2) \times \Psi^*_{k_1\mathbf{P}_1}(\mathbf{R}_1, \mathbf{r}_1) \Psi^*_{k_2\mathbf{P}_2}(\mathbf{R}_2, \mathbf{r}_2) \Psi_{k_3\mathbf{P}_3}(\mathbf{R}_1, \mathbf{r}_1) \Psi_{k_4\mathbf{P}_4}(\mathbf{R}_2, \mathbf{r}_2). \quad (10)$$

The matrix potential of $\hat{U}$ (equation (7)) connecting the states $\langle k_1 = k_2 = 0, \mathbf{P}_1, \mathbf{P}_2|$ and $\langle k_3 = k_4 = 0, \mathbf{P}_3, \mathbf{P}_4|$ has the form

$$\langle \mathbf{P}_1, \mathbf{P}_2|\hat{U}|\mathbf{P}_3, \mathbf{P}_4\rangle = \frac{1}{S^2} U(\mathbf{P}_3 - \mathbf{P}_1) \delta(\mathbf{P}_1 + \mathbf{P}_2 - \mathbf{P}_3 - \mathbf{P}_4), \quad (11)$$

where S is the area of a quantum well, and

$$U(\mathbf{P}_3 - \mathbf{P}_1) = \int \int U(|\mathbf{R}_1 - \mathbf{R}_2|) \exp({\rm i}(\mathbf{P}_3 - \mathbf{P}_1)(\mathbf{R}_1 - \mathbf{R}_2)) {\rm d}^2|\mathbf{R}_1 - \mathbf{R}_2|. \quad (12)$$

In the strong magnetic field limit, using the internal wavefunction of the magnetoexciton in the lowest Landau level for $\Phi_k(\mathbf{P}, \mathbf{r} - \boldsymbol{\rho})$ in equation (4) (recall $\boldsymbol{\rho} = \frac{r_{\rm H}^2}{H}\mathbf{H} \times \mathbf{P}$), we can ignore the variable $\boldsymbol{\rho}$ relative to $\mathbf{r}$ at small magnetic momenta $P \ll 1/r_{\rm H}$. So in $\Psi_{k=0,\mathbf{P}}(\mathbf{R}, \mathbf{r})$ (equation (4)) we put $\mathbf{P} = 0$ and $\boldsymbol{\rho} = 0$ for $\Phi_{k=0}(\mathbf{P}, \mathbf{r} - \boldsymbol{\rho})$, while we keep $\mathbf{PR} \neq 0$ in

the exponent. This procedure is valid in a strong magnetic field at small magnetic momenta, because the characteristic $\boldsymbol{\rho}$ is much smaller than the characteristic $R \sim r_s = (\pi n)^{-1/2}$ ($r_H \ll (\pi n)^{-1/2}$). Keeping $\mathbf{PR} \neq 0$, we can use the magnetic momentum conservation law below. So in the strong magnetic field limit, using for $\Phi_k(\mathbf{0}, \mathbf{r})$ the internal wavefunction of the magnetoexciton in the lowest Landau level [20]

$$\Phi_{k=0}(\mathbf{0}, \mathbf{r}) = \frac{1}{\sqrt{2\pi} r_H} \exp\left[-\frac{r^2}{4r_H^2}\right], \tag{13}$$

we obtain the matrix element of the external potential $V_{e,h}(\mathbf{r})$ connecting the states $\langle k = 0, \mathbf{P}|$ and $\langle k = 0, \mathbf{P}'|$, which is defined as

$$\langle \mathbf{P}'|\hat{V}_{e,h}(\mathbf{r})|\mathbf{P}\rangle = \frac{1}{S}\int d^2R \int d^2r\, V_{e,h}(\mathbf{r}_{e,h})\Psi^*_{k'\mathbf{P}'}(\mathbf{R}, \mathbf{r})\Psi_{k\mathbf{P}}(\mathbf{R}, \mathbf{r}), \tag{14}$$

and has the form

$$\begin{aligned}\langle \mathbf{P}'|\hat{V}_{e,h}(\mathbf{r})|\mathbf{P}\rangle &= \frac{1}{S}\exp\left(-(\mathbf{P}' - \mathbf{P})^2\frac{r_H^2}{4}\right) V_{e,h}(\mathbf{P}' - \mathbf{P}) \exp\left(\pm\frac{ir_H^2}{2H}\mathbf{HP} \times \mathbf{P}'\right) \\ &= \frac{1}{S}\tilde{V}_{e,h}(\mathbf{P}' - \mathbf{P}) \exp\left(\pm\frac{ir_H^2}{2H}\mathbf{HP} \times \mathbf{P}'\right),\end{aligned} \tag{15}$$

where

$$V_{e,h}(\mathbf{P}' - \mathbf{P}) = \int\int V_{e,h}(\mathbf{r}) \exp[i(\mathbf{P}' - \mathbf{P})\mathbf{r}]\, d^2r. \tag{16}$$

Then, using the expressions for the matrix elements (10) and (14), the expansions for the field operators (5), applying the orthonormality of functions $\Phi_k(\mathbf{0}, \mathbf{r})$, and employing only the lowest Landau level $k = 0$ in the strong magnetic field limit, we finally obtain from equation (8) the effective Hamiltonian of dipole indirect magnetoexcitons in high magnetic field in the presence of the disorder, in coordinate space:

$$\begin{aligned}\hat{H}_{\text{eff}} = &\int d\mathbf{R}\hat{\psi}^\dagger(\mathbf{R})\left(-\frac{\nabla^2}{2m_H} + V_{\text{eff}}(\mathbf{R})\right)\hat{\psi}(\mathbf{R}) \\ &+ \tfrac{1}{2}\int d\mathbf{R}_1 \int d\mathbf{R}_2 \hat{\psi}^\dagger(\mathbf{R}_1)\hat{\psi}^\dagger(\mathbf{R}_2)U(\mathbf{R}_1 - \mathbf{R}_2)\hat{\psi}(\mathbf{R}_2)\hat{\psi}(\mathbf{R}_1),\end{aligned} \tag{17}$$

where $\hat{\psi}^\dagger(\mathbf{R})$ and $\hat{\psi}(\mathbf{R})$ are the Bose creation and annihilation field operators (we discuss below the validity of the assumption about magnetoexcitons being bosons), and the coupling to the effective random field $V_{\text{eff}}(\mathbf{R})$ has the form

$$V_{\text{eff}}(\mathbf{R}) = \frac{1}{\pi r_H^2}\int \exp\left(-\frac{(\mathbf{R} - \mathbf{r})^2}{r_H^2}\right)[V_e(\mathbf{r}) + V_h(\mathbf{r})]\, d\mathbf{r}. \tag{18}$$

Equation (18) is valid if the following inequality holds [32]:

$$r_{\text{exc}}\sqrt{\langle \nabla V^2\rangle_{\text{av}}} \ll E_b, \tag{19}$$

and it holds for strong magnetic field, when $r_{\text{exc}} = r_H = (eH)^{-1/2}$, and $E_b \sim e^2/\epsilon D$ at $D \gg r_H$ [20, 30].

The effective magnetoexciton Hamiltonian $\hat{H}_{\text{eff}}$ (17) treats the magnetoexciton as an electrically neutral composite particle. Since the particle is neutral, it does not directly interact with the magnetic field. The interaction with the magnetic field manifests itself through the renormalization of the exciton effective mass and modification of the correlation function of the random field. Thus, we can map the original problem of the dilute weakly interacting magnetoexciton system in a strong magnetic field in the presence of disorder, described by the

total Hamiltonian (1), to the dilute system of excitons without magnetic field with the effective magnetic mass m_{H} and in the presence of the effective random field V_{eff} (equation (18)), which is renormalized by the magnetic field H. The dipole–dipole interaction term in the effective Hamiltonian (17) in the strong magnetic field limit is exactly the same as one for the excitons without magnetic field, because the dipole–dipole interaction does not depend on the relative coordinates of electron and hole in the exciton as long as we assume the excitons are parallel dipoles (as discussed above). This mapping allows us to use the results for the collective spectrum, superfluid density and Kosterlitz–Thouless temperature, obtained for the system in random field without magnetic field [26], for the case of strong magnetic field.

3. The Green's function of a single magnetoexciton in the random field

The interaction between a spatially indirect exciton in coupled quantum wells and a random field, induced by fluctuations in widths of electron and hole quantum wells, has the form [30]:

$$V(\mathbf{r}_{\mathrm{e}}, \mathbf{r}_{\mathrm{h}}) = \alpha_{\mathrm{e}}[\xi_1(\mathbf{r}_{\mathrm{e}}) - \xi_2(\mathbf{r}_{\mathrm{e}})] + \alpha_{\mathrm{h}}[\xi_3(\mathbf{r}_{\mathrm{h}}) - \xi_4(\mathbf{r}_{\mathrm{h}})], \tag{20}$$

where $\alpha_{\mathrm{e,h}} = \partial E^{(0)}_{\mathrm{e,h}}/\partial d_{\mathrm{e,h}}$, where the subscript e or h refers to either the electron or hole, respectively. $d_{\mathrm{e,h}}$ is the average widths of the electron and hole quantum wells, $E^{(0)}_{\mathrm{e,h}}$ are the lowest energy levels of the electron and hole in the conduction and valence bands, and ξ_1 and ξ_2 (ξ_3 and ξ_4) are fluctuations in the widths of the wells on the upper and lower interfaces, respectively. We assume that fluctuations on different interfaces are statistically independent, whereas fluctuations of a specific interface are characterized by the Gaussian correlation function

$$\langle \xi_i(\mathbf{r}_1)\xi_j(\mathbf{r}_2)\rangle = g_i\delta_{ij}\delta(\mathbf{r}_2 - \mathbf{r}_1), \qquad \langle \xi_i(\mathbf{r})\rangle = 0, \tag{21}$$

where g_i is proportional to the squared amplitude of the ith interface fluctuation [30]. This is possible if the distance D between the electron and hole quantum wells is larger than the amplitude of fluctuations on the nearest surfaces.

In order to obtain the Green's function of the magnetoexcitons with dipole–dipole repulsion in the random field, we obtain the Green's function of a single magnetoexciton in the random field (not interacting with other magnetoexcitons), and then apply perturbation theory with respect to the dipole–dipole repulsion between excitons, using the system of the noninteracting magnetoexcitons as a reference system, in analogy with the system without magnetic field [26].

In general, the single-particle Green's function and the kernel of the Bethe–Salpeter equation both depend on the random potential and the Coulomb interaction between the electrons and holes, and, therefore, the disorder and the interactions have to be treated simultaneously. The spectrum of a single noninteracting magnetoexcitons in the presence of disorder was calculated on the level of the Bethe–Salpeter equation in [27]. We consider the case of the correlation length of the random field potential L much shorter than the average distance between excitons $r_{\mathrm{s}} \sim 1/\sqrt{\pi n}$ ($L \ll 1/\sqrt{\pi n}$, where n is the total exciton density); L is also much smaller than the radius of a magnetoexciton ($L \ll r_{\mathrm{exc}}$), and the condition (19) applies. This implies that the imaginary part of the denominator Q is proportional to the random field correlation function strength. Under these assumptions the dipole–dipole repulsion can be treated in the ladder approximation, which implies that the vertex is proportional to the density n, and disorder is treated in the second-order Born approximation. The diagrams contributing to the self-energy containing four lines (two electron and two hole lines), which are responsible for the renormalization of the Coulomb interaction by the random field, are proportional to Qnr^2_{exc}. Since these terms contain the small parameter $nr^2_{\mathrm{exc}} \ll 1$, while the self-energy represented in

our paper contains only terms proportional to Q, these terms can be neglected for a Belyaev weakly interacting Bose gas with the disorder treated by the second-order Born approximation with the negligibly small correlation length of the random field.

Since the effective magnetoexciton Hamiltonian (17) is translationally invariant, we can write the Green's function of an isolated magnetoexciton in momentum space. The Green's function $G^{(0)}(\mathbf{p}, \omega)$ of the center of mass of the isolated magnetoexciton at $T = 0$ in the momentum–frequency domain in the random field in the coherent potential approximation (CPA) is given by [30] (here and below $\hbar = 1$)

$$G^{(0)}(\mathbf{p}, \omega) = \frac{1}{\omega - \varepsilon_0(p) + \mu + \mathrm{i}Q(\mathbf{p}, \omega)}, \tag{22}$$

where μ is the chemical potential of the system, and $\varepsilon_0(p) = p^2/2m_\mathrm{H}$ is the spectrum of the center of mass of the exciton in the 'clean' system. The function $Q(\mathbf{p}, \omega)$ is determined by effective random field acting on the center of mass of the exciton. For zero random field, $Q(\mathbf{p}, \omega) \to 0$. If $\alpha_i^2 g_i m_\mathrm{H} \ll E_\mathrm{b}$, the function $Q(\mathbf{p}, \omega)$ in the coherent potential approximation is given by [30]

$$Q(\mathbf{p}, \omega) = \frac{1}{2} \int G^{(0)}(\mathbf{q}, \omega) B(|\mathbf{p} - \mathbf{q}|) \frac{\mathrm{d}^2 q}{(2\pi)^2}, \tag{23}$$

where

$$B(\mathbf{p}) \equiv \int \mathrm{d}^2 R B(\mathbf{R}) \mathrm{e}^{-\mathrm{i}\mathbf{p}\mathbf{R}}, \tag{24}$$

for $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2$, and in the coordinate domain $B(\mathbf{R}_1, \mathbf{R}_2) = \langle V_\mathrm{eff}(\mathbf{R}_1) V_\mathrm{eff}(\mathbf{R}_2) \rangle_\mathrm{av}$. Using the effective potential V_eff given by equation (18), this has the form [30]

$$B(\mathbf{R}_1, \mathbf{R}_2) = B(\mathbf{R}_1 - \mathbf{R}_2) = \frac{\alpha_\mathrm{e}^2(g_1 + g_2) + \alpha_\mathrm{h}^2(g_3 + g_4)}{2\pi r_\mathrm{H}^2} \exp\left(-\frac{(\mathbf{R}_1 - \mathbf{R}_2)^2}{2r_\mathrm{H}^2}\right). \tag{25}$$

Note that in the limit of strong magnetic fields the magnetic length $r_\mathrm{H} = (eH)^{-1/2}$ is much smaller than the characteristic length of the random field potential L ($r_\mathrm{H} \ll L$), and therefore

$$\lim_{r_\mathrm{H}|\mathbf{R}_1 - \mathbf{R}_2|^{-1} \to 0} B(\mathbf{R}_1 - \mathbf{R}_2) = \frac{\alpha_\mathrm{e}^2(g_1 + g_2) + \alpha_\mathrm{h}^2(g_3 + g_4)}{\sqrt{2\pi} r_\mathrm{H}} \delta(\mathbf{R}_1 - \mathbf{R}_2). \tag{26}$$

The random field acting on a magnetoexciton is represented by the white noise correlation function of the random potential $B(\mathbf{R}_1, \mathbf{R}_2) = \langle V_\mathrm{eff}(\mathbf{R}_1) V_\mathrm{eff}(\mathbf{R}_2) \rangle_\mathrm{av}$. The time-reversal symmetry of the effective magnetoexciton Schrödinger equation in strong magnetic field, corresponding to the effective Hamiltonian (8) with the effective random field (18), and elimination of the long-range property of the random field (equation (26)), cancels all effects related with the broken time-reversal symmetry in the Schrödinger equation (1) for $m_\mathrm{e} \neq m_\mathrm{h}$ that result in corrections to the Green's function of a magnetoexciton [33].

Using equation (24), we obtain the Fourier transform of $B(R)$

$$B(p) = \frac{\alpha_\mathrm{e}^2(g_1 + g_2) + \alpha_\mathrm{h}^2(g_3 + g_4)}{16\pi^4} \exp\left(-\frac{r_\mathrm{H}^2 p^2}{32}\right). \tag{27}$$

Thus the CPA Green's function of the 2D indirect exciton can be determined by the solution of the self-consistent equations (22) and (23).

In the weak scattering limit ($g_i \alpha_i m_\mathrm{H} \ll E_\mathrm{b} \sim e^2/\epsilon D$) we use the second-order Born approximation for Q similar to [29, 30, 26], expanding Q (equation (23)) in series to the first order in $B(|\mathbf{p} - \mathbf{q}|)$ (which is the first order in g_i), and we replace equation (23) by:

$$Q(\mathbf{p}, \omega) = \frac{\pi}{2} \int \delta\left(\omega - \frac{q^2}{2m_\mathrm{H}}\right) B(|\mathbf{p} - \mathbf{q}|) \frac{\mathrm{d}^2 q}{(2\pi)^2}. \tag{28}$$

Substituting $B(p)$ from equation (27) into (28), we obtain for $Q(\mathbf{p}, \omega)$

$$Q(\mathbf{p}, \omega) = \frac{\alpha_e^2(g_1 + g_2) + \alpha_h^2(g_3 + g_4)}{64\pi^4} m_H \exp\left(-\frac{r_H^2}{32}(p^2 + 2m_H\omega)\right) J_0\left(\frac{r_H^2}{16}\sqrt{2m_H\omega}p\right), \tag{29}$$

where $J_0(z)$ is the Bessel first integral. The second-order Born Green function of the single indirect exciton in the random field in CQW $G^{(0)}(\mathbf{p}, \omega)$ is derived by substituting $Q(\mathbf{p}, \omega)$ from equation (29) into (22).

4. Collective spectrum and superfluidity of indirect dirty magnetoexcitons

Due to the interwell separation D, indirect magnetoexcitons both in the ground state and in excited states have electrical dipole moments. We suppose that indirect excitons interact as *parallel* dipoles. This is valid when D is larger than the mean separation $\langle r \rangle$ between electron and hole in the magnetoexciton along quantum wells $D \gg \langle r \rangle$. We approximate that at high magnetic fields $\langle r \rangle \approx P r_H^2$ ($\langle \mathbf{r} \rangle$ is normal to $\mathbf{P}$) and that the typical value of magnetic momenta is $P \sim \sqrt{n_{ex}}$ (with exactness to the logarithm of the exciton density $\ln(n_{ex})$, see below), if the dispersion relation $\varepsilon_k(P) = \frac{P^2}{2m_{Hk}}$ is true. Therefore the inequality $D \gg \langle r \rangle$ is valid when $D \gg \sqrt{n} r_H^2$.

The distinction between excitons and bosons manifests itself in exchange effects (see [34, 35, 13]). These effects for excitons with spatially separated e and h in a dilute system $n_{ex}a^2(H, D) \ll 1$ are suppressed due to the negligible overlapping of wavefunctions of two excitons on account of the potential barrier, associated with the dipole–dipole repulsion of indirect excitons [13] (here n_{ex}, $a(D, H)$ are the exciton density and the magnetoexciton radius along quantum wells, respectively). The small tunneling parameter connected with this barrier is

$$\exp\left[-\int_{a(H,D)}^{r_0} \sqrt{2m_{Hk}\left(\frac{e^2D^2}{R^3} - \frac{\kappa^2}{2m_{Hk}}\right)}\,\mathrm{d}R\right], \qquad \text{where } \kappa^2 \sim n/\ln(1/8\pi n m_{Hk}^2 e^4 D^4)$$

is the characteristic momentum of the system (see below); $r_0 = (2m_{Hk}e^2D^2/\kappa^2)^{1/3}$ is the classical turning point for the dipole–dipole potential at the energy equal to the chemical potential of the system. In high magnetic fields the small parameter mentioned above has the form $\exp[-2(m_{Hk})^{1/2}eDa^{-1/2}(H, D)]$. Therefore the system of indirect magnetoexcitons can be treated by the diagram technique for a boson system.

Since the effective Hamiltonian $\hat{H}_{eff}$ (equation (17)) of the system of indirect 'dirty' magnetoexcitons at small momenta is exactly identical to the Hamiltonian of indirect 'dirty' excitons without magnetic field, replacing the excitonic mass $M = m_e + m_h$ by the magnetic mass m_H and using the effective random field V_{eff}, we can use the expressions for the ladder approximation Green's function [36, 37], collective spectrum, normal and superfluid density and the temperature of Kosterlitz–Thouless phase transition [25] for the 'dirty' system without magnetic field [26], replacing the excitonic mass and Q. Since the characteristic length of the random field potential L is much shorter than the average distance between electron and hole, the first step in the averaging procedure with respect to the random field results in the Green's function of a noninteracting exciton with an imaginary part. The second step takes care of the exciton–exciton repulsion. The results of [26] show that the density of the superfluid component and the temperature of the phase transition decrease with increasing disorder, and give good correspondence with the results of the Lopatin–Vinokur approach for the weakly interacting bosons in the presence of disorder [38].

Since the characteristic frequencies and momenta which give the greatest contribution to the Green's function in the ladder approximation are [37] $\omega\epsilon D/e^2 \sim n/\log[r_{\rm H}^2/(nD^4)] \ll 1$ and $pr_{\rm H} \sim m_{\rm H}\sqrt{n/\log[r_{\rm H}^2/(nD^4)]} \ll 1$, respectively, for the single-exciton Green's function $G^{(0)}(\mathbf{p},\omega)$, participating in the ladder approximation, we approximate $Q(\mathbf{p},\omega)$ by $Q(\mathbf{p}=\mathbf{0},\omega=0)$ (see equation (29)):

$$Q(\mathbf{p},\omega) = Q = \frac{\alpha_{\rm e}^2(g_1+g_2)+\alpha_{\rm h}^2(g_3+g_4)}{64\pi^4}m_{\rm H}. \tag{30}$$

The chemical potential μ is obtained in the form (compare to [26]):

$$\mu = \frac{\kappa^2}{2m_{\rm H}} = \frac{8\pi n}{2m_{\rm H}\log\left(\frac{\epsilon^2}{8\pi n m_{\rm H}^2 e^4 D^4}\right)} \tag{31}$$

where κ is a characteristic momentum. The condensate Green's function $D(\mathbf{p},{\rm i}\omega_k)$ is (cf [26])

$$D^{(0)}(\mathbf{p},{\rm i}\omega_k) = \frac{-{\rm i}(2\pi)^2 n_0\delta(\mathbf{p})}{{\rm i}\omega_k+\mu+{\rm i}Q}, \tag{32}$$

where n_0 is the density of Bose condensate. Since at small temperatures $(n-n_0)/n \ll 1$, according to the ladder approximation [36] we use below n instead of n_0. $G(\mathbf{p},{\rm i}\omega_k)$ and $F(\mathbf{p},{\rm i}\omega_k)$ are the normal and anomalous Green functions of the noncondensate:

$$\begin{aligned} G(\mathbf{p},{\rm i}\omega_k) &= -\frac{{\rm i}\omega_k+\varepsilon_0(p)+\mu+{\rm i}Q}{\omega_k^2+\varepsilon^2(p)-2{\rm i}(\mu-\varepsilon_0(p))Q}; \\ F(\mathbf{p},{\rm i}\omega_k) &= -\frac{\mu}{\omega_k^2+\varepsilon^2(p)-2{\rm i}(\mu-\varepsilon_0(p))Q}, \end{aligned} \tag{33}$$

where $\varepsilon_0(p)$ is the spectrum of noninteracting excitons; the spectrum of interacting excitons has the form

$$\varepsilon(p) = \sqrt{(p^2/(2m_{\rm H})+\sqrt{\mu^2-Q^2})^2-(\mu^2-Q^2)},$$

and for small momenta $p \ll \sqrt{2m_{\rm H}\mu}$, the excitation spectrum is acoustic $\varepsilon(p) = c_{\rm s}p$, where $c_{\rm s} = \sqrt{\sqrt{\mu^2-Q^2}/m_{\rm H}}$ is the velocity of sound.

At large magnetic momenta P the isolated magnetoexciton spectrum $\varepsilon(P)$, contrary to the case $H=0$, has a constant limit, equal to the Landau level $\frac{\omega_{\rm c}}{2}$ for the reduced effective mass (see [20, 14]). As a result, the spectrum of the interacting magnetoexciton system also has a plateau at high momenta. Therefore, the Landau criterion of superfluidity is not strictly valid at large P for the interacting magnetoexciton system. However, the probability of excitation of quasiparticles with magnetic momenta $P \gg 1/r_{\rm H}$ by a moving magnetoexciton system is negligibly small at small superfluid velocities [21]. In this sense, the appearance of a linear branch can be taken as the criterion for superfluidity of 2D magnetoexcitons.

The density of the superfluid component $n_{\rm s}(T)$ can be obtained using the relation $n_{\rm s}(T) = n - n_{\rm n}(T)$, where $n_{\rm n}(T)$ is the density of the normal component. The density of normal component $n_{\rm n}$ is (cf [26]):

$$n_{\rm n} = n_{\rm n}^0 + \frac{N}{m_{\rm H}}\int\frac{{\rm d}\mathbf{p}}{(2\pi)^2}p^2\mu\frac{\varepsilon_0(p)}{\varepsilon^4(p)}Q. \tag{34}$$

Here N is the total number of particles, and $n_{\rm n}^0$ is the density of the normal component in a pure system with no impurities,

$$n_{\rm n}^0 = -\frac{1}{2m_{\rm H}}\int\frac{{\rm d}\mathbf{p}}{(2\pi)^2}p^2\frac{\partial n_0(p)}{\partial\varepsilon}, \tag{35}$$

where $n_0(p) = (e^{\varepsilon(p)/T} - 1)^{-1}$ is the distribution of an ideal Bose gas of temperature excitations.

The first term in equation (34), which does not depend on Q, is the contribution to the normal component due to scattering of quasiparticles with an acoustic spectrum in a pure system at $T \neq 0$ (cf [26]),

$$n_n^0 = \frac{3\zeta(3)}{2\pi} \frac{T^3}{c_s^4(n, Q)m_H}, \tag{36}$$

where $\zeta(z)$ is the Riemann zeta function ($\zeta(3) \simeq 1.202$). The second term in equation (34) is the contribution to the normal component due to the interaction of the particles (magnetoexcitons) with the random field,

$$n_n = n_n^0 + \frac{nQ}{2m_H c_s^2(n, Q)} = \frac{3\zeta(3)}{2\pi} \frac{T^3}{c_s^4(n, Q)m_H} + \frac{nQ}{2m_H c_s^2(n, Q)}. \tag{37}$$

The density of the superfluid component is $n_s = n - n_n$. From equations (36) and (37) we can see that the random field decreases the density of the superfluid component.

In a 2D system, superfluidity appears below the Kosterlitz–Thouless transition temperature $T_c = \pi n_s/(2m_H)$ [25], where only coupled vortices are present. Using the expressions (36) and (37) for the density n_s of the superfluid component, we obtain an equation for the Kosterlitz–Thouless transition temperature T_c. Its solution is

$$T_c = \left[\left(1 + \sqrt{\frac{32}{27} \left(\frac{m_H T_c^0}{\pi n'} \right)^3 + 1} \right)^{1/3} - \left(\sqrt{\frac{32}{27} \left(\frac{m_H T_c^0}{\pi n'} \right)^3 + 1} - 1 \right)^{1/3} \right] \frac{T_c^0}{2^{1/3}}. \tag{38}$$

Here T_c^0 is an auxiliary quantity, equal to the temperature at which the superfluid density vanishes in the mean-field approximation $n_s(T_c^0) = 0$,

$$T_c^0 = \left(\frac{2\pi n' c_s^4 m_H}{3\zeta(3)} \right)^{1/3}. \tag{39}$$

In equations (38) and (39), n' is

$$n' = n - \frac{nQ}{2m_H c_s^2}. \tag{40}$$

Even though an expression similar to equation (38) has been presented earlier in [26], the result of this paper is nontrivial because it takes into account the influence of the strong magnetic field on the excitonic superfluidity using mapping of the system with magnetic field onto the system without field by replacing excitonic mass by magnetic mass.

5. Discussion

Figures 1 and 2 show the dependence of the Kosterlitz–Thouless critical temperature on magnetic field and particle density, for different cases of the disorder potential. The superfluid density n_s and the temperature of the Kosterlitz–Thouless transition T_c to the superfluid state for the 'clean', zero-disorder system at fixed exciton density n decrease as the magnetic mass m_H increases [21] (we can see this by setting $Q = 0$, which makes $n' = n$ and $c_s = \sqrt{\mu/m_H}$ in (38)). Since in strong magnetic fields, when $D \gg r_H$, the exciton magnetic mass is $m_H \approx D^3\epsilon/(e^2 r_H^4)$ [20], the superfluid density n_s and the temperature of the Kosterlitz–Thouless transition T_c decrease with increase of the magnetic field H. There is an additional effect due to the influence of the magnetic field on the effective random field felt by the

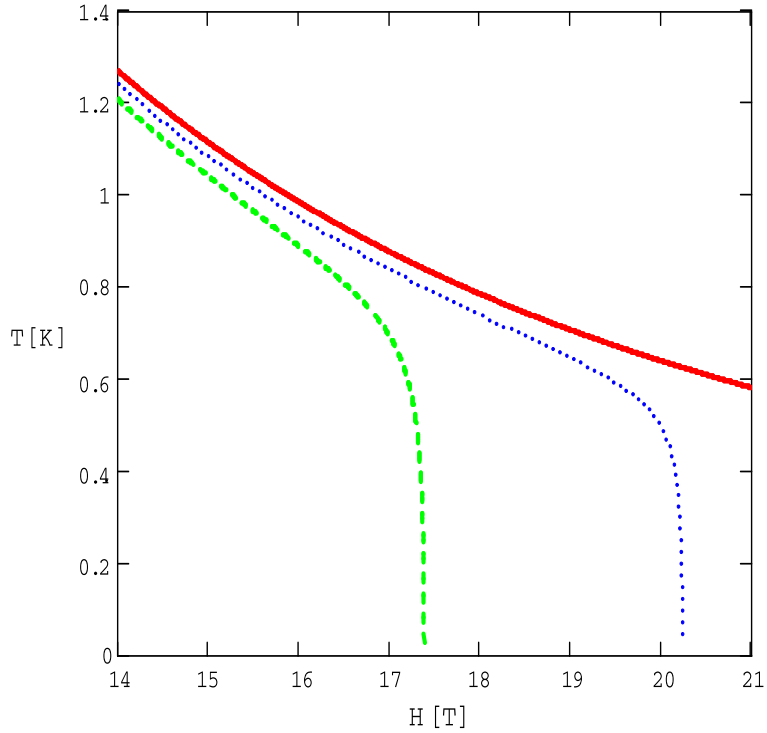

Figure 1. Dependence of temperature of Kosterlitz–Thouless transition $T_c = T_c(H)$ (in units K; for GaAs/GaAsAl: $M = 0.24m_0$; $\epsilon = 13$; m_0 is a mass of electron) on the magnetic field H (in units T) at $g_i = 10^4$ nm^4 at the interwell distance $D = 15$ nm; at the exciton density $n = 1.0 \times 10^{11}$ cm^{-2}; at the different parameters of the random field $\alpha = \alpha_e = \alpha_h$: $\alpha = 0$—solid curve; $\alpha = 0.5$ meV nm^{-1}—dotted curve; $\alpha = 0.7$ meV nm^{-1}—dashed curve.

excitons. Since in 'dirty' systems, n_s and T_c decrease with increase of the effective random field parameter Q (analogous to the case without magnetic field, see [26]), and in a strong magnetic field Q is proportional to m_H (equation (30)), the increase of the magnetic field H increases the effective renormalized random field Q, and suppresses the superfluid density n_s and the temperature of the Kosterlitz–Thouless transition T_c. Note that the approximation used slightly underestimates the role of the disorder. So the estimation of the renormalized random field Q, the suppression of n_s an T_c are also slightly underestimated (the influence of the approximation will be discussed in detail elsewhere).

The quantum phase transition presented in figures 1 and 2 is equivalent to the superfluid–Bose glass transition in disordered bosonic systems. The latter transition has been extensively discussed in literature (see [39–41]). Since the overlapping integral of the magnetoexcitonic wavefunctions is proportional to the factor of $\exp[-\langle r\rangle/r_s]$ (where the average electron–hole separation in the plane parallel to quantum wells $\langle r\rangle$ is the magnetoexcitonic radius, and $r_s \sim (\pi n)^{-1/2}$ is the average distance between two magnetoexcitons), for the very dilute systems with the small parameter $\langle r\rangle/r_s \ll 1$ the composite nature of the magnetoexcitons can be neglected, and the quantum phase transition is equivalent to the superfluid to Bose glass transition in the system of 'dirty' noncomposite bosons presented in [39–41].

Note that in the presence of the disorder there is a quantum transition from a superfluid state to one in which the superfluid state is completely suppressed, even at zero temperature,

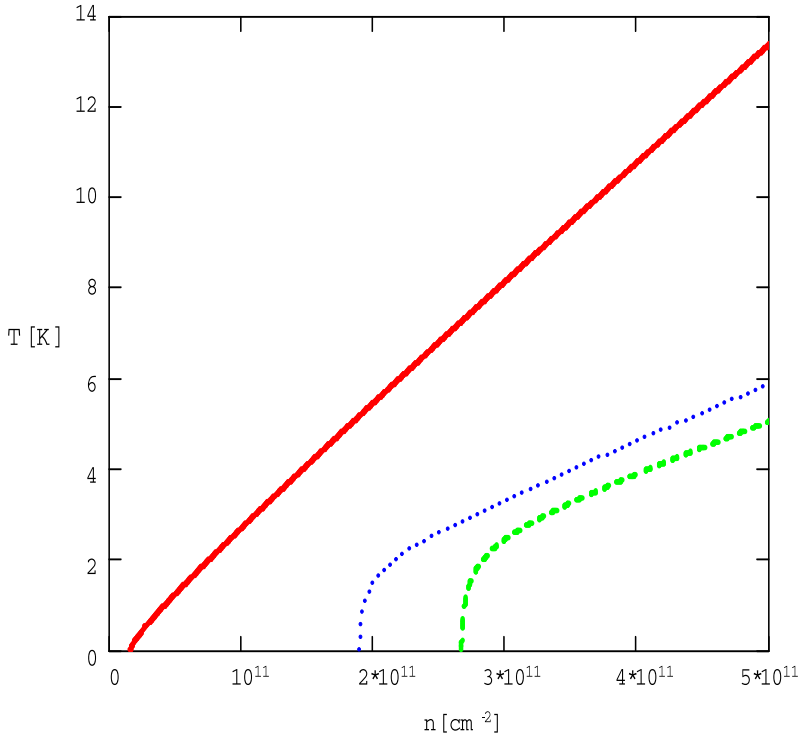

Figure 2. Dependence of temperature of Kosterlitz–Thouless transition $T_c = T_c(n)$ (in units K; for GaAs/GaAsAl: $M = 0.24m_0$; $\epsilon = 13$; m_0 is a mass of electron) on the exciton density n (in units cm^{-2}) at the interwell distance $D = 15$ nm at the parameters of the random field $\alpha_e = \alpha_h = 1.5$ meV nm^{-1}; $g_i = 10^4$ nm^4; at different magnetic fields H (in units of T): $H = 0$—solid curve (for $H = 0$ we use the results of [30] for spin degeneracy factor $s = 4$); $H = 14$ T—dotted curve; $H = 15$ T—dashed curve.

as the magnetic field H is varied. While in the 'clean' system at any magnetic field H there is always a region in the density–temperature space where the superfluidity occurs [21], in the presence of the disorder at sufficiently large magnetic field H and large enough parameters of the disorder α_e, α_h and g_i, there is no superfluidity at any exciton density.

Note also that in a magnetic field the superfluid density n_s and the temperature of the Kosterlitz–Thouless transition T_c decrease when the separation between quantum wells D increases, because n_s and T_c are decreasing functions of the magnetoexciton effective mass m_H (equations (37) and (38)), which is an increasing function of D [20]. The dependence of n_s and T_c on D through the magnetic mass $m_H \approx D^3\epsilon/(e^2 r_H^4)$ and through the dependence on the random field Q, which increases with increasing m_H (equation (30)), is stronger than their increasing logarithmic dependence on D through the velocity of sound related to the logarithmic dependence on D of the chemical potential of the dipole–dipole repulsion (equation (31)). The latter results in n_s and T_c increasing with D in the case without magnetic field [26]. The decrease of n_s and T_c with increasing D at high magnetic fields H is significant when D is large ($D \sim 30r_H$), which is far from the typical D used in experiments, because at very large values of D the superfluid system of magnetoexcitons must transform into the system of two incompressible liquids [19], i.e. the binding energy of the magnetoexcitons vanishes in the limit $D \to \infty$.

In the high magnetic field limit at high D, the effective random field is not small, and the approaches assuming coupling with the random field to be much smaller than the dipole–dipole repulsion [42, 43] are not applicable. Note that in the present work the parameter Q/μ is not required to be small, and our formulae for the superfluid density and Kosterlitz–Thouless temperature can be used in the regime of realistic experimental parameters taken from photoluminescence line broadening measurements [5]. The coherent potential approximation (CPA) allows us to derive the exciton Green's function for the wide range of the random field, and in the weak scattering limit the CPA results in the second-order Born approximation.

The system of 'dirty' indirect magnetoexcitons can appear also in unbalanced two-layer *electron* systems in CQW in strong magnetic fields near the filling factor $\nu = 1$. An external electric voltage between quantum wells changes the filling, so, for example, in the first quantum well the filling factor can be $\nu_1 = \Delta\nu \ll \frac{1}{2}$ and in another one it will be $\nu_2 = 1 - \Delta\nu$. Unbalanced filling factors $\nu_1 = 1 + \Delta\nu$, $\nu_2 = 1 - \Delta\nu$ are also possible. Thus in the first quantum well (QW) there are dilute electrons on the second Landau level, and in the second QW there are dilute empty places (holes) in the first Landau level. Excess electrons in the first QW and holes in the second QW can bind to indirect magnetoexcitons with the density $n_{\rm ex} = eH\Delta\nu/2\pi$. The influence of disorder on the superfluidity in two-layer unbalanced e–e system in high magnetic fields is completely analogous to the one for two-layer e–h system.

Note that in this paper we assumed spin degeneracy factor $s = 1$ in high magnetic fields H. Since at $H = 0$ the spin degeneracy is $s = 4$ [26], at low H lifting spin degeneracy will have substantial effect on the magnetoexciton superfluidity, which we do not take into account in this paper where we consider only high H.

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References

[1] Butov L V, Zrenner A, Abstreiter G, Bohm G and Weimann G 1994 *Phys. Rev. Lett.* **73** 304
Butov L V and Filin A I 1998 *Phys. Rev.* B **58** 1980
[2] Denev S, Negoita V, Snoke D W, Laikhtman B, Eberl K and Pfeiffer L 2002 *Phys. Rev.* B **66** 205304
[3] For a general review of experiments in quantum wells, see Moskalenko S A and Snoke D W 2000 *Bose–Einstein Condensation of Excitons and Biexcitons and Coherent Nonlinear Optics with Excitons* (New York: Cambridge University Press) chapter 10
[4] Levitov L S, Simons B D and Butov L V 2005 *Phys. Rev. Lett.* **94** 176404
[5] Snoke D, Denev S, Liu Y, Pfeiffer L and West K 2002 *Nature* **418** 754
[6] Snoke D 2002 *Science* **298** 1368
[7] Lozovik Yu E and Yudson V I 1976 *Zh. Eksp. Teor. Fiz.* **71** 738
Lozovik Yu E and Yudson V I 1976 *JETP* **44** 389 (Engl. Transl.)
[8] Zimmermann R 1987 *Many-Particle Theory of Highly Excited Semiconductors* (Leipzig: Teubner)
[9] Zhu X, Littlewood P B, Hybertsen M S and Rice T M 1995 *Phys. Rev. Lett.* **74** 1633
[10] Keeling J, Levitov L S and Littlewood P B 2004 *Phys. Rev. Lett.* **92** 176402
[11] Littlewood P B, Eastham P R, Keeling J, Marchetti F M, Simons B D and Szymanska M H 2004 *J. Phys.: Condens. Matter* **16** S3597
[12] Conti S, Vignale G and MacDonald A H 1998 *Phys. Rev.* B **57** R6846
[13] Lozovik Yu E, Berman O L and Willander M 2002 *J. Phys.: Condens. Matter* **14** 12457
[14] Lerner I V and Lozovik Yu E 1978 *Zh. Eksp. Teor. Fiz.* **74** 274
Lerner I V and Lozovik Yu E 1978 *JETP* **47** 146 (Engl. Transl.)

Lerner I V and Lozovik Yu E 1979 *Zh. Eksp. Teor. Fiz.* **76** 1136
Lerner I V and Lozovik Yu E 1979 *JETP* **49** 576 (Engl. Transl.)
[15] Paquet D, Rice T M and Ueda K 1985 *Phys. Rev.* B **32** 5208
[16] Lerner I V and Lozovik Yu E 1980 *Zh. Eksp. Teor. Fiz.* **78** 1167
Lerner I V and Lozovik Yu E 1980 *JETP* **51** 588 (Engl. Transl.)
Lerner I V and Lozovik Yu E 1981 *Zh. Eksp. Teor. Fiz.* **80** 1488
Lerner I V and Lozovik Yu E 1981 *JETP* **53** 763 (Engl. Transl.)
[17] Balatsky A V, Joglekar Y N and Littlewood P B 2004 *Phys. Rev. Lett.* **93** 266801
[18] Kallin C and Halperin B I 1984 *Phys. Rev.* B **30** 5655
Kallin C and Halperin B I 1985 *Phys. Rev.* B **31** 3635
[19] Yoshioka D and MacDonald A H 1990 *J. Phys. Soc. Japan* **59** 4211
[20] Lozovik Yu E and Ruvinsky A M 1997 *Zh. Eksp. Teor. Fiz.* **112** 1791
Lozovik Yu E and Ruvinsky A M 1997 *JETP* **85** 979 (Engl. Transl.)
[21] Lozovik Yu E, Berman O L and Tsvetus V G 1997 *Pis. Zh. Eksp. Teor. Fiz.* **66** 332
Lozovik Yu E, Berman O L and Tsvetus V G 1999 *Phys. Rev.* B **59** 5627
[22] Voros Z, Balili R, Snoke D W, Pfeiffer L and West K 2005 *Phys. Rev. Lett.* **94** 226401
[23] Olivares-Robles M A and Ulloa S E 2001 *Phys. Rev.* B **64** 115302
[24] Moskalenko S A, Liberman M A, Snoke D W and Botan V V 2002 *Phys. Rev.* B **66** 245316
[25] Kosterlitz J M and Thouless D J 1973 *J. Phys. C: Solid State Phys.* **6** 1181
Nelson D R and Kosterlitz J M 1977 *Phys. Rev. Lett.* **39** 1201
[26] Berman O L, Lozovik Yu E, Snoke D W and Coalson R D 2004 *Phys. Rev.* B **70** 235310
[27] Grochol M, Grosse F and Zimmermann R 2005 *Phys. Rev.* B **71** 125339
Grochol M, Grosse F and Zimmermann R 2005 *J. Lumin.* **112** 208
[28] Lozovik Yu E, Berman O L and Ruvinsky A M 1999 *Pis. Zh. Eksp. Teor. Fiz.* **69** 573
Lozovik Yu E, Berman O L and Ruvinsky A M 1999 *JETP Lett.* **69** 616 (Engl. Transl.)
[29] Gevorkyan Zh S and Lozovik Yu E 1985 *Fiz. Tverd. Tela* **27** 1800
Gevorkyan Zh S and Lozovik Yu E 1985 *Sov. Phys. Solid State* **27** 1079 (Engl. Transl.)
[30] Lozovik Yu E and Ruvinsky A M 1998 *Zh. Eksp. Teor. Fiz.* **114** 1451
Lozovik Yu E and Ruvinsky A M 1998 *JETP* **87** 788 (Engl. Transl.)
[31] Gor'kov L P and Dzyaloshinskii I E 1967 *Zh. Eksp. Teor. Fiz.* **53** 717
Gor'kov L P and Dzyaloshinskii I E 1968 *JETP* **26** 449 (Engl. Transl.)
[32] Bonch-Bruevich V L, Zvyagin I P, Kaĭper R, Mironov A G, Énderlaĭn R and Ésser B 1981 *Electronic Theory of Disordered Semiconductors* (Moscow: Nauka) p 350
[33] Arseyev P I and Dzyubenko A B 1995 *Phys. Rev.* B **52** R2261
[34] Keldysh L V and Kozlov A N 1968 *Zh. Eksp. Teor. Fiz.* **54** 978
Keldysh L V and Kozlov A N 1968 *JETP* **27** 521 (Engl. Transl.)
[35] Zimmermann R 1970 *Phys. Status Solidi* **37** K133
Zimmermann R 1970 *Phys. Status Solidi* **39** 95
[36] Abrikosov A A, Gor'kov L P and Dzyaloshinskii I E 1963 *Methods of Quantum Field Theory in Statistical Physics* (Englewood Cliffs, NJ: Prentice-Hall)
[37] Lozovik Yu E and Yudson V I 1978 *Physica* A **93** 493
[38] Lopatin A V and Vinokur V M 2002 *Phys. Rev. Lett.* **88** 235503
[39] Ma M, Halperin B I and Lee P A 1986 *Phys. Rev.* B **34** 3136
[40] Fisher M P A, Weichman P B, Grinstein G and Fisher D S 1989 *Phys. Rev.* B **40** 546
[41] Gunn J M F and Lee D K K 1989 *Physica* C **162** 1483
Lee D K K and Gunn J M F 1990 *Physica* B **165** 509
Lee D K K and Gunn J M F 1990 *J. Phys.: Condens. Matter* **2** 7753
Lee D K K and Gunn J M F 1992 *J. Low Temp. Phys.* **89** 101
[42] Huang K 1995 *Bose–Einstein Condensation* ed A Griffin, D W Snoke and S Stringari (Cambridge: Cambridge University Press) pp 31–50
[43] Huang K and Meng H F 1992 *Phys. Rev. Lett.* **69** 644
Meng H F 1994 *Phys. Rev.* B **49** 1205